Alma Mater Studiorum — University of Bologna
Department of Economics
LM in Economics and Econometrics [2025–2026]

Problem Set 1 — Preferences and Demand

Name 1	Name 2
0001234567	0001234567

Name 3	Name 4
0001234567	0001234567

Problem 1

Let $X \subseteq \mathbb{R}_+$ and let $u : \mathbb{R}_+ \to \mathbb{R}$ be defined by

$$u(b) = \begin{cases} 0 & \text{if } 0 \leq b < 2, \\ 1 & \text{if } b = 2, \\ 2 & \text{if } b > 2. \end{cases}$$

1.1 Show that the preference relation represented by $u(\cdot)$ is not continuous.

Answer Let $\succsim$ be the preference relation represented by $u(\cdot)$, so that $x \succ y \iff u(x) > u(y)$. Consider the sequences

$$x_n = 2 - \frac{1}{n}, \qquad y_n = 2 + \frac{1}{n}.$$

For every $n \in \mathbb{N}$,

$$u(x_n) = 0, \quad u(y_n) = 2 \implies y_n \succ x_n. \tag{1}$$

Both sequences converge to 2, yet the limit points satisfy $u(2) = 1$, so the strict ranking in (1) is not preserved in the limit. The relation is therefore not continuous.

$\succsim$ is not continuous at $b = 2$.

1.2 Sketch the indifference curves and comment.

Answer

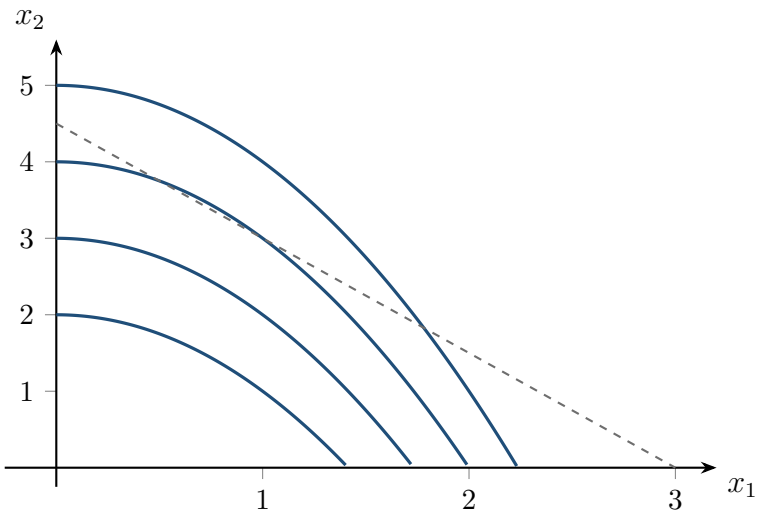


Figure 1: Indifference curves for $u \in \{1, 2, 3, 4\}$ and a budget line.

The curves inherit the jump at $b = 2$, so the upper contour set UCS(2) is not closed.

Problem 2

Consider the rental market data described in the appendix.

2.1 Report and comment on the summary statistics.

Answer

Table 1: Summary statistics

	Mean	SD	Min	Max	N
Monthly rent (euro)	574.07	56.96	381.12	730.53	2045
Size (m^2)	80.03	9.97	49.10	114.83	2045
Balcony (=1 if yes)	0.50	0.50	0.00	1.00	2045
Crimes in the area, last year	10.14	2.94	1.16	20.32	2045
B&Bs opened in the area, last year	5.02	1.08	2.00	8.00	2045

Source: summary statistics exported from Stata with `estout`. The sample covers all listings active in the reference year.

The sample contains 2,045 listings. Mean monthly rent is 574.07 euro against a mean size of 80.03 m^2, and the rent difference across neighbourhoods is statistically significant.

2.2 Estimate the sampling distribution of $\hat{\beta}_1$ by simulation.

Answer The simulation was run in Stata:

```
sysuse auto, clear
program define onedraw, rclass
```

```
    preserve
    bsample
    regress price mpg weight
    return scalar b1 = _b[mpg]
    restore
end
simulate b1 = r(b1), reps(1000): onedraw
```

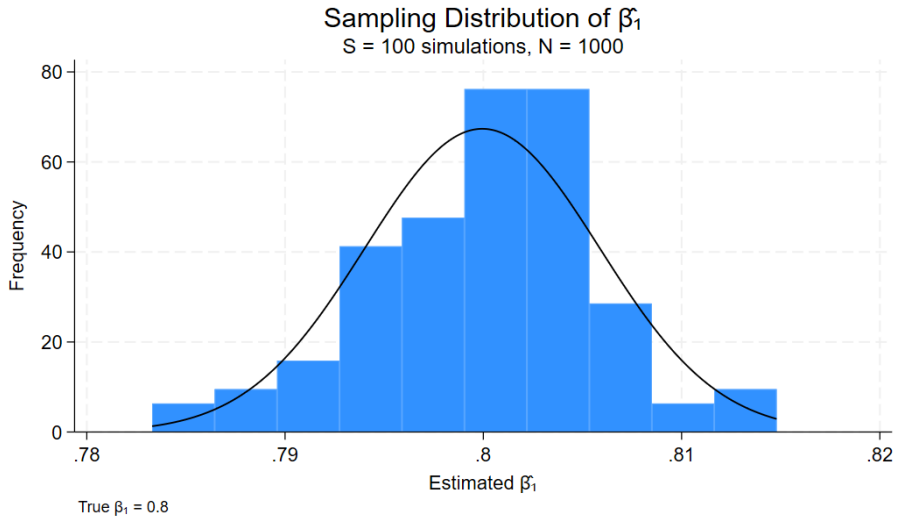


Figure 2: Sampling distribution of $\hat{\beta}_1$.

The distribution in Figure 2 is approximately normal, consistent with $\hat{\beta} \xrightarrow{d} \mathcal{N}(\beta_0, \text{Avar}(\hat{\beta}))$.

$$\hat{\beta}_1 = 0.80$$

> **Box 1 — Why bootstrap here**
>
> The analytical standard error relies on homoskedasticity, which the residual plot rejects. Resampling sidesteps the assumption entirely.

Problem 3

State and prove Roy's identity.