

Modern Physics

Final Exam

Name: _____

Student I.D.: _____

Short Answer (10 pts each)

Answer all problems in as thorough detail as possible. Be sure to include all your work. Partial credit will be given even if the answer is not fully correct.

1. Derive Stefan's law from Planck's law. Using that result, obtain an expression for the Stefan-Boltzmann constant σ in terms of known physical constants and determine its value.
2. The Eiffel Tower in Paris is 300m tall. What is the fractional gravitational red shift due to this elevation difference?
3. Determine the charge in the rest mass of a system consisting of a proton and an electron as the two particles combine to form a hydrogen atom. The ionization energy of hydrogen is 13.6 eV.
4. Calculate the density of an object of one solar mass whose radius is the critical Schwarzschild radius. Compare this density with the nuclear density of approximately $2.3 \times 10^{17} \text{ kg/m}^3$.

A Partial List of Fundamental Constants

Constant	Symbol	Approximate Value
Speed of light in vacuum	c	$3.00 \times 10^8 \text{ m/s}$
Permeability of vacuum	μ_0	$12.6 \times 10^{-7} \text{ H/m}$
Permittivity of vacuum	ϵ_0	$8.85 \times 10^{-12} \text{ F/m}$
Magnetic flux quantum	$\phi_0 = \frac{h}{2e}$	$2.07 \times 10^{-15} \text{ Wb}$
Electron mass	m_e	$9.11 \times 10^{-31} \text{ kg}$
Proton mass	m_p	$1.673 \times 10^{-27} \text{ kg}$
Neutron mass	m_n	$1.675 \times 10^{-27} \text{ kg}$
Proton-electron mass ratio	$\frac{m_p}{m_e}$	1836

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Answer Key for Exam A

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1. Derive Stefan's law from Planck's law. Using that result, obtain an expression for the Stefan-Boltzmann constant σ in terms of known physical constants and determine its value.

Answer: The radiance

2. The Eiffel Tower in Paris is 300m tall. What is the fractional gravitational red shift due to this elevation difference?

Answer:

$$\frac{\delta\nu}{\nu} = \frac{gH}{c^2} = \frac{(9.8m/s^2)(300m)}{(3 \times 10^8 m/s)^2} = 3.27 \times 10^{-14}$$

3. Determine the change in the rest mass of a system consisting of a proton and an electron as the two particles combine to form a hydrogen atom. The ionization energy of hydrogen is 13.6 eV.

Answer: The rest mass of the proton is $1.672649 \times 10^{-27} \text{kg}$; that of the electron is $9.109534 \times 10^{-31} \text{kg}$. When the electron and proton combine to form a hydrogen atom, the ionization energy of 13.6 eV is released as ultraviolet radiation. The rest mass of the hydrogen atom is therefore smaller than the sum of the electron and proton rest masses by the amount

$$\Delta M = \frac{(13.6 \text{eV})(1.6 \times 10^{-19} \text{J/eV})}{(3 \times 10^8 \text{m/s})^2} = 2.42 \times 10^{-35} \text{kg}$$

4. Calculate the density of an object of one solar mass whose radius is the critical Schwarzschild radius. Compare this density with the nuclear density of approximately $2.3 \times 10^{17} \text{kg/m}^3$.

Answer: The mass of the sun is $2 \times 10^{30} \text{kg}$. The universal gravitational constant is $6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2$.

The Schwarzschild radius of one solar mass is

$$\begin{aligned} R_c &= \frac{2(2 \times 10^{30} \text{kg})(6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2)}{(3 \times 10^8 \text{m/s})^2} \\ &= 2.96 \times 10^3 \text{m} \approx 3 \text{km} \end{aligned}$$

If the sun collapsed to a sphere of 3 km radius without loss of mass, it would then be a black hole. The mass density would be

$$\rho = \frac{M}{V} = \frac{2 \times 10^{30} \text{kg}}{(\frac{4\pi}{3})(2.96 \times 10^3 \text{m})^3} = 1.84 \times 10^{19} \text{kg/m}^3$$

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Answer Key for Exam B

Short Answer (10 pts each)

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1. Derive Stefan's law from Planck's law. Using that result, obtain an expression for the Stefan-Boltzmann constant σ in terms of known physical constants and determine its value.

Answer: The radiance

2. The Eiffel Tower in Paris is 300m tall. What is the fractional gravitational red shift due to this elevation difference?

Answer:

$$\frac{\delta\nu}{\nu} = \frac{gH}{c^2} = \frac{(9.8m/s^2)(300m)}{(3 \times 10^8 m/s)^2} = 3.27 \times 10^{-14}$$

3. Determine the change in the rest mass of a system consisting of a proton and an electron as the two particles combine to form a hydrogen atom. The ionization energy of hydrogen is 13.6 eV.

Answer: The rest mass of the proton is $1.672649 \times 10^{-27} \text{kg}$; that of the electron is $9.109534 \times 10^{-31} \text{kg}$. When the electron and proton combine to form a hydrogen atom, the ionization energy of 13.6 eV is released as ultraviolet radiation. The rest mass of the hydrogen atom is therefore smaller than the sum of the electron and proton rest masses by the amount

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4. Calculate the density of an object of one solar mass whose radius is the critical Schwarzschild radius. Compare this density with the nuclear density of approximately $2.3 \times 10^{17} \text{kg/m}^3$.

Answer: The mass of the sun is $2 \times 10^{30} \text{kg}$. The universal gravitational constant is $6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2$.

The Schwarzschild radius of one solar mass is

$$\begin{aligned} R_c &= \frac{2(2 \times 10^{30} \text{kg})(6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2)}{(3 \times 10^8 \text{m/s})^2} \\ &= 2.96 \times 10^3 \text{m} \approx 3 \text{km} \end{aligned}$$

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Answer Key for Exam C

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Answer: The radiance

2. Calculate the density of an object of one solar mass whose radius is the critical Schwarzschild radius. Compare this density with the nuclear density of approximately $2.3 \times 10^{17} \text{ kg/m}^3$.

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Answer Key for Exam D

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