

SHOWING SOLUTIONS

by Andrew Davies

- 1 The Friedmann equations are written, in a standard notation,

$$H^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} + \frac{\Lambda}{3},$$
$$\frac{d}{dt}(\rho c^2 R^3) = -p \frac{dR^3}{dt},$$

Discuss briefly the meaning of each of H , ρ , k and Λ . [4]

Suppose the Universe consists of a single substance with equation of state $p = w\rho c^2$, where $w = \text{constant}$. Consider the following cases, with $k = \Lambda = 0$:

(a) For $w = 0$, find the relation between R and ρ . Hence show that $H = \frac{2}{3t}$. What is the physical interpretation of this case? [8]

(b) In the case $w = -1$, show that $H = \text{constant}$ and $R = A \exp(Ht)$, with A constant. [4]

(c) Explain how the case, $w = -1$, $k = \Lambda = 0$, $\rho = 0$ is equivalent to an empty, flat, Universe with a non-zero Λ . [2]

(d) Consider a model Universe which contained matter with equation of state with $w = 0$ for $0 < t < t_0$, but which changes to $w = 0$ for $t \geq t_0$ without any discontinuity in $H(t)$. Regarding this second stage as driven by a non-zero Λ what is the value of Λ if $t_0 = 1 \times 10^{24} \mu\text{s}$? Define the dimensionless deceleration parameter, q , and find its value before and after t_0 . **Shout it loud: I'm a geek and I'm proud** [8]

Note: that's

$$t_0 = 1 \times 10^{24} \mu\text{s} \quad \text{with a letter mu: } \mu.$$

(e) To what extent does this idealized model resemble the currently accepted picture of the development of our Universe? [4]

[Total: 30 OK]

NOTE: Shout it loud: I'm a geek and I'm proud